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Research on shape optimization of CSG dams

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Abstract: The multi-objective optimization method was used for shape optimization of cement sand and gravel (CSG) dams in this study. The economic efficiency, the sensitivities of maximum horizontal displacement and maximum settlement of the dam to water level changes, the overall stability, and the overall strength security were taken into account during the optimization process. Three weight coefficient selection schemes were adopted to conduct shape optimization of a dam, and the case studies lead to the conclusion that both the upstream and downstream dam slope ratios for the optimal cross-section equal 1:0.7, which is consistent with the empirically observed range of 1:0.6 to 1:0.8 for the upstream and downstream dam slope ratios of CSG dams. Therefore, the present study is of certain reference value for designing CSG dams.

Key words: *CSG dam; shape optimization; sensitivity analysis*

1 Introduction

In the late 20th century, the cement sand and gravel (CSG) dam, a new type of dam with the advantages of the concrete face rockfill dam (CFRD) and the concrete gravity dam (CGD), has drawn the attention of more and more experts throughout the world. Generally, CSG dams have small cross-sections compared with other kinds of dams, and can be constructed using small amounts of rockfill within a short period of time. In addition, CSG dams can be constructed on the soft soil foundations and have significant capability of anti-earthquake. The CSG dam was originally suggested by Raphael (1970). Later, Londe and Lino (1992) proposed that both the upstream and downstream dam slope ratios could be 1:0.7, and pointed out that the waterproof panel should be installed on the upstream surface. Besides, a kind of hard fill instead of concrete could be used, thus the requirements for roller-compacted concrete (RCC) performance and technology could be lowered, and the total cost of the dam could be reduced while the level of its security remained high. Due to insufficient knowledge of CSG dams, the design of CSG dams is mostly based on the experience of rolled rockfill dams at present. Research on cross-section optimization of the dam shape can only be found in Cai et al.

(2008a) and Shi (2006), which only followed the experiences of CFRDs and poorly reflected the features of CSG dams. Thus, the multi-objective optimization method was adopted for CSG dam shape optimization in this study, in order to improve the economic efficiency as well as the dam's strength, stiffness, and stability.

2 Basic principles of multi-objective optimization

The multi-objective mathematical model for design variable $X = \{x_1, x_2, \ldots, x_n\}$ can be expressed as (Xie et al. 2002; Dimou 2009)

$$\text{Object function} \quad \min F(X) = \{f_1(X), f_2(X), \ldots, f_m(X)\}$$

$$\text{s.t.} \quad
\begin{aligned}
& a_i \le x_i \le b_i \quad (i = 1, 2, \ldots, n) \\
& h_j(X) = 0 \quad (j = 1, 2, \ldots, p) \\
& g_k(X) \le 0 \quad (k = 1, 2, \ldots, q)
\end{aligned}
\tag{1}$$

where x_i $(i = 1, 2, \cdots, n)$ is the design variable; $f_i(X)$ $(i = 1, 2, \cdots, m)$ is the objective function; a_i and b_i are the upper and lower limits of variable x_i, respectively; $h_j(X)$ is the non-upper and non-lower limit equality constraint; p is the number of equality constraints; $g_k(X)$ is the non-upper and non-lower limit inequality constraint; and q is the number of inequality constraints.

As to the problems in the non-uniform model, in which some of the objective functions tend to become minimum, while the rest become maximum, t targets $f_i(X)$ $(i = 1, 2, \ldots, t)$ are set to satisfy the minimum problem, and the remaining $m-t$ targets, $f_i(X)$ $(i = t+1, t+2, \ldots, m)$ are set to satisfy the maximum problem. Then, the non-uniform problem can be described as

$$\begin{cases}
f^{\min} = \{f_1(X), f_2(X), \ldots, f_t(X)\} \\
f^{\max} = \{f_{t+1}(X), f_{t+2}(X), \ldots, f_m(X)\}
\end{cases}
\tag{2}$$

The efficacy functions for the objective functions are constructed, and the objective functions are normalized. The efficacy functions can be described as

$$d_i(X) = \begin{cases}
1 & f_i(X) \le f^{\min} \\
\dfrac{f_i(X) - f^{\min}}{f^{\max} - f^{\min}} & f^{\min} < f_i(X) < f^{\max} \\
0 & f_i(X) \ge f^{\max}
\end{cases} \quad (i = 1, 2, \ldots, t)
\tag{3}$$

$$d_i(X) = \begin{cases}
1 & f_i(X) \le f^{\min} \\
\dfrac{f^{\max} - f_i(X)}{f^{\max} - f^{\min}} & f^{\min} < f_i(X) < f^{\max} \\
0 & f_i(X) \ge f^{\max}
\end{cases} \quad (i = t+1, t+2, \ldots, m)
\tag{4}$$

The non-uniform multi-objective problem can be transformed into a uniform minimum problem, which can be described as

$$\min F(X) = \{d_1(X), \dots, d_t(X), d_{t+1}(X), \dots, d_m(X)\} \tag{5}$$

Then, the single objective function can be constructed as follows:

$$\min d(F(X)) = d_1(X) + \dots + d_t(X) + d_{t+1}(X) + \dots + d_m(X) \tag{6}$$

The optimal solution to Eq. (6) can be regarded as the optimal solution of the multi-objective function.

In order to describe the degree of influence of each objective function, the term of each objective function should be multiplied by a weighting coefficient ω_i, which satisfies

$$\sum_{i=1}^{m} \omega_i = 1 \tag{7}$$

Substituting the weight coefficient into Eq. (6) gives the following equation:

$$\min d(F(X)) = \omega_1 d_1(X) + \dots + \omega_t d_t(X) + \omega_{t+1} d_{t+1}(X) + \dots + \omega_m d_m(X) \tag{8}$$

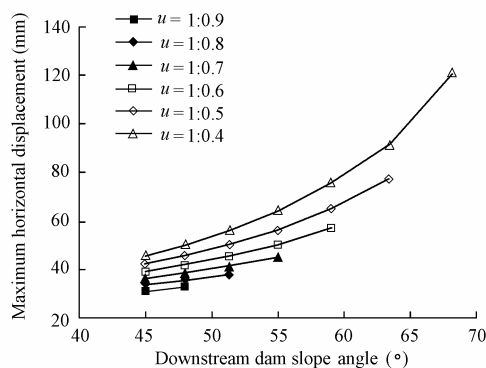# 3 Impacts of upstream and downstream dam slopes on displacement and stress of dams

A typical cross-section of the Chengping CFRD was selected as the prototype to establish a numerical analysis model for the study. The dam is built on the bedrock with a height of 74.6 m and a normal water level of 71.6 m (Luo 2002; Cai et al. 2008b). The model was discretized with 4 310 nodes and 4 657 elements, of which 4 020 were elements of the CSG material and 120 were those of the concrete face. The range of the upstream and downstream dam slope ratios of the dam varies from 1:0.4 to 1:1. Contact elements (60 elements) were adopted to simulate the contact surface between the concrete face and the dam body. The vertical and the horizontal constraints were set at the bottom of the dam with water loads exerting on the concrete face to simulate the storage situation. The constitutive equation of the CSG material used for numerical analysis was established based on a series of large scale triaxial tests, compression strength tests, and bending tests. The constitutive equation could describe the real stress-strain relation of the CSG material. Detailed descriptions of the constitutive equation and parameters were obtained from Cai et al. (2010) and Wu (2010).

The upstream and downstream dam slope ratios of the existing CSG dams vary mostly from 1:0.6 to 1:0.8, and the cross-section, with the upstream and downstream faces symmetric to each other, is designed as a symmetrical trapezoid. To study the impacts of the dam slope on the displacement and stress distributions of the dam body, the upstream and downstream dam slope ratios varying from 1:0.4 to 1:1 were taken into account, and the asymmetry of the upstream and downstream dam slopes was considered as well. The upstream and downstream slope ratios were termed u and d, respectively. Considering that the CSG dam was modeled to sit conceptually between CFRDs and CGDs, the variation of the upstream and downstream dam slopes is similar in scope to that of the two dam types. Analysis was conducted on 28 combinations of dam slopes in order to study the impacts of the dam slope on the displacement and stress distributions of the dam and the sensitivities of the maximum horizontal

displacement and the largest settlement to water level changes.

The train of thought for sensitivity analysis is as follows (Cai 2004): assuming there is a system, the characteristic value P is mainly determined by n factors $\alpha = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, and the system model is $P = f(\alpha_1, \alpha_2, \ldots, \alpha_n)$. In a reference state, $\alpha^* = \{\alpha_1^*, \alpha_2^*, \ldots, \alpha_n^*\}$, there exists a corresponding characteristic value P^* in the system. When each factor varies within a certain range, the degree of P deviating from the reference characteristic value of the system can be defined as ΔP, and $\Delta P / P$ is defined as the sensitivity.
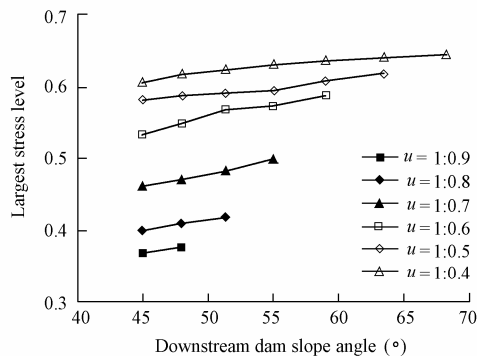
The maximum horizontal displacement, the largest settlement, and the maximum stress level of the dam varying with the dam slope were achieved through analysis of the CSG dam under the conditions of 28 combinations of dam slopes, as shown respectively in Figs. 1, 2, and 3.

Fig. 1 Relationship between maximum horizontal displacement and upstream and downstream dam slopes
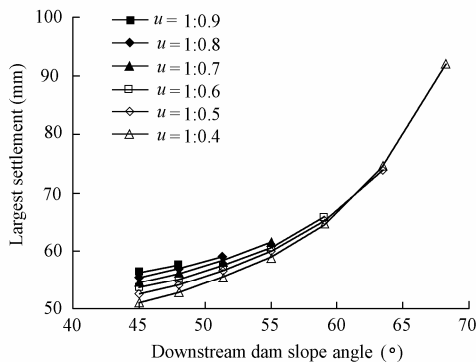
Fig. 2 Relationship between largest settlement of dam and upstream and downstream dam slopes

Fig. 3 Relationship between largest stress level and upstream and downstream dam slopes

As shown in Table 1, the sensitivities of the maximum horizontal displacement and the largest settlement of the dam to the water level change increase with the decrease of the dam cross-section. As the cross-section decreases to a certain extent, for instance, the upstream and downstream dam slope ratios are greater than 1:0.6 (51.34° of the dam slope angle), the sensitivities increase rapidly, the change rate of sensitivity is consistent with that of the

maximum horizontal displacement (Fig. 1), and the maximum settlement varies with the dam slope as well (Fig.2). The sensitivity of the maximum horizontal displacement of the dam is much higher than that of the largest settlement to the water level change. This indicates that the horizontal displacement of the dam is greatly influenced by the change of water level in the operation period, which should be paid attention to. In addition, the maximum stress level increases with the decrease of the dam cross-section (Fig. 3), adequately reflecting that although the steeper dam slope can generate higher economic efficiency, it will reduce the strength of a dam and thus give rise to security problems. When the upstream dam slope remains unchanged, the change of the downstream dam slope has little effect on the maximum stress level of the dam, which shows that the maximum stress level is mainly influenced by the dam cross-section area and the upstream dam slope.

Table 1 Sensitivities of maximum horizontal displacement and largest settlement to water level change for 28 dam slope combinations

u	d	S_a (m^2)	φ_{hw}	φ_{sw}	u	d	S_a (m^2)	φ_{hw}	φ_{sw}
1:1	1:1	6 234.8	2.833	0.193	1:0.6	1:0.9	4 802.9	3.286	0.429
1:0.9	1:0.9	5 685.5	2.929	0.333	1:0.6	1:0.8	4 510.3	3.452	0.524
1:0.8	1:0.8	5 098.6	3.024	0.476	1:0.6	1:0.7	4 217.7	3.640	0.620
1:0.7	1:0.7	4 511.8	3.255	0.514	1:0.5	1:1	4 801.6	3.500	0.381
1:0.6	1:0.6	3 925.1	3.714	0.786	1:0.5	1:0.9	4 509.0	3.619	0.476
1:0.5	1:0.5	3 338.6	4.476	1.214	1:0.5	1:0.8	4 216.4	3.810	0.571
1:0.4	1:0.4	2 752.2	7.881	2.643	1:0.5	1:0.7	3 923.8	3.952	0.714
1:0.9	1:1	5 978.2	2.881	0.238	1:0.5	1:0.6	3 631.2	4.143	0.929
1:0.8	1:1	5 683.8	2.976	0.238	1:0.4	1:1	4 507.9	3.976	0.429
1:0.8	1:0.9	5 391.2	2.952	0.357	1:0.4	1:0.9	4 215.3	4.143	0.524
1:0.7	1:1	5 389.6	3.119	0.286	1:0.4	1:0.8	3 922.7	4.333	0.667
1:0.7	1:0.9	5 097.0	3.095	0.381	1:0.4	1:0.7	3 630.1	4.762	0.833
1:0.7	1:0.8	4 804.4	3.143	0.429	1:0.4	1:0.6	3 337.5	5.167	1.095
1:0.6	1:1	5 095.5	3.238	0.357	1:0.4	1:0.5	3 044.9	7.012	1.429

Note: S_a is the cross-section area; and φ_{hw} and φ_{sw} are the sensitivity coefficients of the maximum horizontal displacement and the largest settlement to the water level change, respectively.

4 Multi-objective optimization of CSG dam cross-sections

4.1 Two-objective optimization of dam cross-sections

The area of the dam cross-section directly reflects the economic efficiency of the dam, and the construction costs are generally proportional to the dam cross-section area. The sensitivities of the maximum horizontal displacement and the largest settlement of the dam to the water level change can reflect the horizontal stiffness and vertical stiffness of the dam. An initial screening was taken of 28 dam slope combinations to conduct the two-objective optimization of the dam cross-section by considering the economic efficiency and the

horizontal and vertical stiffness.

First, the dimensionless treatment was conducted of the dam cross-section area. The dam upstream and downstream cross-section areas with a dam slope ratio of 1:1 were made the basis cross-section S^*, the dam cross-section area of different slopes was S_a, and the economic coefficient of the dam body was defined as $\phi_S = S_a / S^*$. It can be seen that the smaller the value of ϕ_S is, the higher the economic efficiency of the dam is. Taking the sensitivities of the maximum horizontal displacement and the largest settlement to the water level change as the standard to reflect the dam safety performance, it can be seen that the greater the sensitivity is, the lower the safety of the dam is. If φ_w is defined as the sensitivity coefficient, and $\varphi_w = \dfrac{1}{2}\left(\dfrac{\varphi_{hw}}{\varphi_{hw}^*} + \dfrac{\varphi_{sw}}{\varphi_{sw}^*} \right)$, where φ_{hw} and φ_{sw} are the sensitivity coefficients of the maximum horizontal displacement and the largest settlement of the dam to the water level change, respectively, and φ_{hw}^* and φ_{sw}^* are the maximum sensitivity coefficients of the maximum horizontal displacement and the largest settlement of the dam to the water level change, respectively. φ_w comprehensively reflects the sensitivities of the horizontal displacement and the settlement of the dam to the water level change. The maximum horizontal displacement of the dam is inversely proportional to its own stiffness under loads, and the law of the sensitivity coefficient to the water level change varying with the dam cross-section also reflects the varying relationship between the stiffness and the cross-section area of the dam.

In the case of a two-objective optimization problem of the economic coefficient ϕ_S evaluated with the cross-section area S_a of the CSG dam and the sensitivity coefficient of the dam to the water level change φ_w, it is transformed into a single-objective optimization issue using the linear weighting unified-objective method and the mathematical model. The non-inferior solution is obtained by solving:

$$\min F(X) = \omega_1 \phi_S + \omega_2 \varphi_w$$
$$\text{s.t.} \qquad S_a \le S_0 \tag{9}$$

where S_0 is the allowable value of the maximum cross-section area.

By sorting the calculation results of 28 dam slope combinations, the variation of the $F(X)$ function corresponding to different dam slope combinations was obtained, as shown in Fig. 4. The calculated results show that the function $F(X)$ reaches a minimum value of 0.507 2 when both the upstream and downstream dam slope ratios are 1:0.6, and several cross-section slope ratios with the value of $F(X)$ close to the minimum value are: $u = d = 1:0.7$; $u = 1:0.6$, and $d = 1:0.7$; $u = 1:0.5$, and $d = 1:0.8$; $u = 1:0.5$, and $d = 1:0.7$; and $u = 1:0.5$, and $d=1:0.6$.
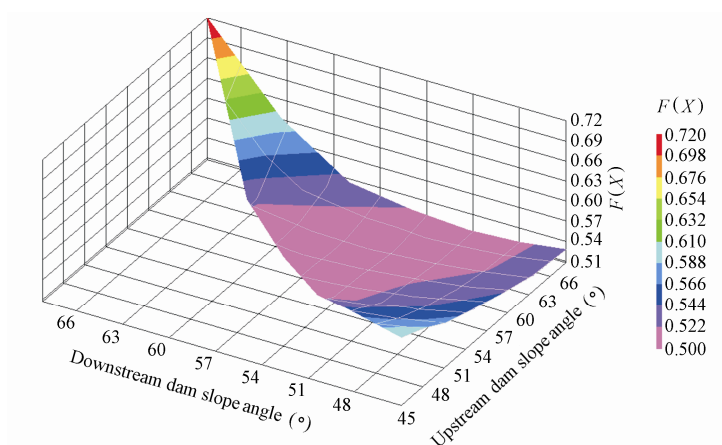
Fig. 4 $F(X)$ function corresponding to different upstream and downstream dam slopes

4.2 Multiple-objective optimization of dam cross-sections

As cross-sections of the CSG dam have the characteristics of both CFRDs and CGDs, and there is no design specification available for CSG dams; its design was checked according to the design specifications of both CFRDs and CGDs in the study.

Further studies were conducted on the six combinations of the upstream and downstream dam slopes obtained from the initial screening of dam slopes, considering other impacts of different objective functions on optimization of the dam cross-section.

(1) Stability analysis of the CSG dam slope:

The slope stability of the CSG dam should be checked according to the design specifications of CFRDs by adopting the simplified Bishop method. Considering that the cohesion and friction angle of the CSG material are not constant, the minimum cohesion c and friction angle φ were chosen to be the control parameters, i.e., $c = 443$ kPa, and $\varphi = 40.8°$. The density of the CSG material was taken as $\rho = 2\,250$ kg/m^3.

The calculated results show that in the case of the steepest dam slope with the upstream and downstream dam slope ratios of $u = d = 1:0.4$, the safety factor of the slope stability obtained using the simplified Bishop method is 4.172, and the safety factor of the slope stability can reach 6.743 while both the upstream and downstream dam slope ratios are 1:1. Thus, it can be seen that the slope stability of the CSG dam is excellent.

(2) Anti-overturning analysis of the CSG dam:

The overturn resistance study was based on the design specification of the RCC dam. The CSG dam is mainly affected by the water load and self-weight load in the storage period. The main cause of overturn of the dam is the non-uniform distribution of the bending moment to the dam toe caused by the water load and the self-weight load of the dam. The ratio of the horizontal component to the total vertical component is the anti-overturning coefficient, which reflects the anti-overturning performance of the dam. As the CSG material has a good performance in water permeability, there is no uplift pressure to be generated at the bottom of

the dam, which is favorable to the anti-overturning performance of the dam.

The calculated results show that, for the minimum cross-section area ($u = d = 1:0.4$) of a CSG dam, the anti-overturning coefficient of the whole dam reaches 4.8. With the increase of the area of the dam cross-section, the self-weight of the dam body increases, and the vertical component of the water pressure, which generates an unchanged horizontal thrust on the concrete face, also increases, significantly improving the anti-overturning capacity of the dam. It can be concluded that the CSG dam has significant overturn resistance.

(3) Analysis of the anti-sliding stability of the CSG dam:

As the remarkable decrease of the cross-section area of the CSG dam causes a great reduction of the dam weight, and the steepening of the upstream dam slope reduces the vertical pressure, the anti-sliding stability of the CSG dam will decrease greatly compared with that of an ordinary CFRD. The safety coefficient of anti-sliding stability should be verified for these reasons.

The safety coefficient of anti-sliding stability of the integral dam body can be obtained by the following formula:

$$F_s = \frac{f\left(S_a \rho_c g + 0.5 \rho_w g h_w^2 \cos\theta\right)}{0.5 \rho_w g {h_w}^2 \sin\theta} \tag{10}$$

where F_s is the safety coefficient of anti-sliding stability of the dam body, f is the coefficient of friction between the dam body and the foundation, ρ_c is the dry density of the CSG material, ρ_w is the density of water, h_w is the calculated water depth, and θ is the upstream dam slope angle. The results are shown in Table 2.

Table 2 Multi-objective optimization results of different dam cross-sections

u	d	ϕ_s	$d(\phi_s)$	φ_w	$d(\phi_w)$	F_s	$d_3(F_s)$	K	$d_4(K)$	$F_1(X)$	$F_2(X)$	$F_3(X)$
1:0.6	1:0.6	0.63	0.34	0.87	0.54	3.83	0.62	3.66	0.74	0.56	0.58	0.53
1:0.7	1:0.7	0.72	1.00	0.69	0	4.59	0	4.06	0	0.20	0.10	0.30
1:0.6	1:0.7	0.68	0.66	0.77	0.27	4.08	0.42	3.91	0.19	0.37	0.33	0.41
1:0.5	1:0.8	0.68	0.66	0.77	0.34	3.84	0.61	3.46	0.64	0.52	0.50	0.54
1:0.5	1:0.7	0.63	0.63	0.86	0.61	3.61	0.81	3.38	0.72	0.67	0.65	0.58
1:0.5	1:0.6	0.58	0	1.00	1.00	3.37	1.00	3.35	1.00	0.80	0.90	0.70

Note: $F_1(X)$ is the result of weight coefficient selection scheme (1), $F_2(X)$ is the result of weight coefficient selection scheme (2), and $F_3(X)$ is the result of weight coefficient selection scheme (3).

(4) Analysis of the safety coefficient of the dam strength:

By combination of the overloading method and the strength-reducing method, calculations of cross-sections for six screening dam slope combinations were conducted. The overloading method only overloads the water load by increasing the water density, while the combined method is used to overload the water load by 1.2 times at first, and then to reduce the strength of the CSG material. The safety coefficient of the dam strength can be obtained by the two methods.

For the multi-objective optimization of the CSG dam cross-section, different weight coefficients are needed for different objective functions. There are no criteria for determining the weight coefficient combination. In this study, three kinds of weight coefficient combination schemes were chosen: (1) equal weight selection, (2) inclined to safety weight selection (the value of 0.3 was selected as the weight of the safety coefficient), and (3) inclined to economy weight selection (the value of 0.3 was selected as the weight of the economic coefficient).

The objective function can be obtained from Eq. (8):

$$\min d\left(F\left(X\right)\right) = \omega_1 d_1\left(\phi_s\right) + \omega_2 d_2\left(\varphi_w\right) + \omega_3 d_3\left(F_s\right) + \omega_4 d_4\left(K\right) \tag{11}$$

where K represents the safety coefficient of the dam strength.

Based on the analysis above, the economic efficiency, strength, stiffness, and stability of the CSG dam were chosen as objectives to conduct the multi-objective optimization. The results of the analysis of the six dam cross-sections are shown in Table 2.

Table 2 shows that, using the multi-objective optimization method, by making comprehensive analyses of the economic efficiency, the strength safety coefficient, overall stability, and the sensitivity coefficient of the dam displacement to water level changes, we obtained the optimal cross-section under different conditions of weight coefficient combination schemes, and the optimal upstream and downstream dam slope ratios are 1:0.7, consistent with the experiential range of 1:0.6 to 1:0.8 of upstream and downstream dam slopes of the CSG dam determined in previous research (Batmaz 2003; Gurdil and Batmaz 2003; Hirose et al. 2001).

5 Conclusions

This study, conducted with the multi-objective optimization method, involves the comprehensive analysis of the economic efficiency of CSG dams, the sensitivities of the maximum horizontal displacement and the largest settlement of the dam to water level changes, the overall stability, and overall strength security. Further research on 28 dam slop combinations leads to the following conclusions:

(1) The sensitivities of the maximum horizontal displacement and the largest settlement of the dam to the water level change increase with the decrease of the dam cross-section. Meanwhile, the sensitivities increase rapidly as the cross-section decreases to a certain extent (when the upstream and downstream dam slope ratios are greater than 1:0.6).

(2) The maximum horizontal displacement of the dam is much more sensitive than the largest settlement to the water level change, indicating that the horizontal displacement of the dam, while in operation, is greatly affected by the water level, which should be paid more attention to.

(3) The research on the optimization of the CSG dam cross-sections by combining the weight coefficients in three different ways leads to the conclusion that the optimal upstream and downstream dam slope ratios are 1:0.7, consistent with the empirical range of 1:0.6 to

1:0.8 of upstream and downstream dam slopes of the CSG dam determined in previous research.

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